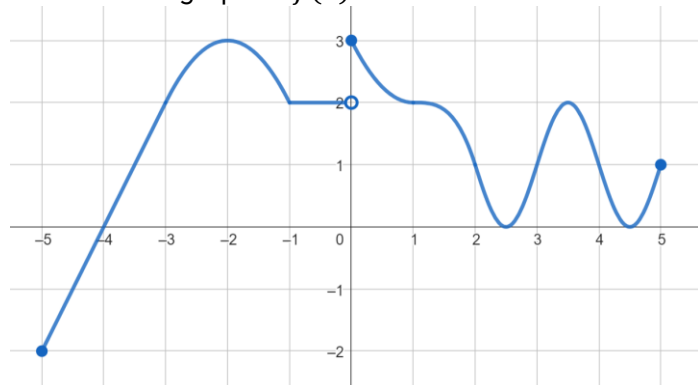


Composition of Functions (2.7) Homework

Use the following graph of $f(x)$, table of $g(x)$ values, and equation of $h(x)$ to find the value of the expression in #1-6 and also to answer #7 and #8.

A graph of $f(x)$ is shown below.



A table of select values of $g(x)$ is given below.

x	$g(x)$
-1	3
0	-2
1	4
2	1
3	0
4	5

Let $h(x) = x + 2$

- $(f \circ g)(3)$
- $(g \circ f)(3)$
- $g(h(0))$
- $h(f(-3))$
- $(f \circ h)(-3)$
- $g(g(1))$
- Describe the transformation that maps the graph of $f(x)$ to the graph of $f(h(x))$.
- Describe the transformation that maps the graph of $f(x)$ to the graph of $h(f(x))$.
- Valentina runs an Etsy shop selling custom aprons. If an apron takes x minutes to produce, Valentina charges her customers $f(x)$ dollars. If the apron is x inches long, it takes Valentina $g(x)$ minutes to produce it. If Valentina charges x dollars for an apron, she makes $h(x)$ dollars in profit after taxes/cost of materials/shipping. If a customer orders an apron that is x inches long, which of the following calculates Valentina's profit?
 - $f(g(h(x)))$
 - $f(h(g(x)))$
 - $h(f(g(x)))$
 - $h(g(f(x)))$
 - $g(f(h(x)))$
 - $g(h(f(x)))$
- Let $f(x) = 3x + 2$ and let $g(x) = \frac{x-1}{x+1}$. Which of the following functions is equivalent to $(g \circ f)(x)$?
 - $\frac{3x+1}{3x+3}$
 - $\frac{3x-3}{x+1} + 2$
 - $\frac{3x+1}{x+1}$
 - $\frac{3x-3}{x+1} + 6$

11. Let $f(x) = x^2$ and let $g(x) = 2x - 1$.

a. Solve $f(g(x)) = 49$

b. Solve $g(f(x)) = 49$

12. Ida is thinking of a function $f(x)$. If $f(g(x)) = g(x)$ and $g(f(x)) = g(x)$ for all x . Which of the following is true?

(A) $f(x) = g(x)$

(B) $f(x) = -g(x)$

(C) $f(x) = \frac{1}{g(x)}$

(D) $f(x) = x$

$$f(x) = x + 4$$

$$g(x) = 4x$$

$$k(x) = -x$$

Let f , g , and h be the functions given above. Let $r(x)$ be a function that is purposely not given in this question. Match each of the following numbered equations with the letter that describes its graph.

13. $r(g(f(x)))$

A) The graph of this function is the image of the graph of $r(x)$ after a reflection over the y -axis followed by a vertical dilation by a factor of 4.

14. $f(r(k(x)))$

B) The graph of this function is the image of the graph of $r(x)$ after a horizontal dilation by a factor of $\frac{1}{4}$ followed by a vertical translation by 4 units.

15. $k(g(r(x)))$

C) The graph of this function is the image of the graph of $r(x)$ after a reflection over the y -axis followed by horizontal translation by 4 units.

16. $f(r(g(x)))$

D) The graph of this function is the image of the graph of $r(x)$ after vertical dilation by a factor of 4 followed by a reflection over the x -axis.

17. $r(f(k(x)))$

E) The graph of this function is the image of the graph of $r(x)$ after a reflection over the y -axis followed by a vertical translation by 4 units.

18. $g(r(k(x)))$

F) The graph of this function is the image of the graph of $r(x)$ after a horizontal dilation by factor $\frac{1}{4}$ followed by a horizontal translation by -4 units.

19. Let $f(x) = \sqrt{x-1}$ and let $g(x) = x^2$

a. Find the domain of $(f \circ g)(x)$

b. Find the domain of $(g \circ f)(x)$

Composition of Functions (2.7) HW Answers

1. $f(g(3)) = f(0) = 3$
2. $g(f(3)) = g(1) = 4$
3. $g(h(0)) = g(2) = 1$
4. $h(f(-3)) = h(2) = 4$
5. $f(h(-3)) = f(-1) = 2$
6. $g(g(1)) = g(4) = 5$
7. $f(h(x)) = f(x + 2)$. This is a horizontal translation of the graph of $f(x)$ by -2 units.
8. $h(f(x)) = f(x) + 2$. This is a vertical translation of the graph of $f(x)$ by 2 units.
9. $g(x)$ inputs length and outputs time. $f(x)$ inputs time and outputs price. $h(x)$ inputs price and outputs profit.

Correct answer: (C) $h(f(g(x))) = h(f(g(\text{length}))) = h(f(\text{time})) = h(\text{price}) = \text{profit}$

$$10. g(f(x)) = g(3x + 2) = \frac{(3x+2)-1}{(3x+2)+1} = \frac{3x+1}{3x+3}$$

Correct answer: (A)

11.

$$a. f(g(x)) = f(2x - 1) = (2x - 1)^2$$

$$\begin{aligned} f(g(x)) &= 49 \\ (2x - 1)^2 &= 49 \\ 2x - 1 &= 7 \text{ or } 2x - 1 = -7 \\ x &= 4 \text{ or } x = -3 \end{aligned}$$

$$b. g(f(x)) = g(x^2) = 2x^2 - 1$$

$$\begin{aligned} g(f(x)) &= 49 \\ 2x^2 - 1 &= 49 \\ 2x^2 &= 50 \\ x^2 &= 25 \\ x &= -5 \text{ or } x = 5 \end{aligned}$$

12. D, since $f(x)$ has no effect on $g(x)$, $f(x)$ is the identity function.

13. F since $r(g(f(x))) = r(4(x + 4))$

14. E since $f(r(k(x))) = r(-x) + 4$

15. D since $k(g(r(x))) = -4r(x)$

16. B since $f(r(g(x))) = r(4x) + 4$

17. C since $r(f(k(x))) = r(-x + 4) = r(-(x - 4))$

18. A since $g(r(k(x))) = 4r(-x)$

19.

$$a. (-\infty, -1] \cup [1, \infty)$$

$$b. [1, \infty)$$